OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Core Mathematics 3

4723

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes

* 4 7 2 3 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find

(i) $\int 8e^{-2x}\,dx$,

(ii) $\int (4x + 5)^6\,dx$. **[5]**

2 **(i)** Use Simpson's rule with four strips to find an approximation to

$$\int_4^{12} \ln x\,dx,$$

giving your answer correct to 2 decimal places. **[4]**

(ii) Deduce an approximation to $\int_4^{12} \ln(x^{10})\,dx$. **[1]**

3 **(i)** Express $2\tan^2\theta - \dfrac{1}{\cos\theta}$ in terms of $\sec\theta$. **[3]**

(ii) Hence solve, for $0° < \theta < 360°$, the equation

$$2\tan^2\theta - \frac{1}{\cos\theta} = 4.$$ **[4]**

4 For each of the following curves, find $\dfrac{dy}{dx}$ and determine the exact x-coordinate of the stationary point:

(i) $y = (4x^2 + 1)^5$, **[3]**

(ii) $y = \dfrac{x^2}{\ln x}$. **[4]**

5 The mass, M grams, of a certain substance is increasing exponentially so that, at time t hours, the mass is given by

$$M = 40e^{kt},$$

where k is a constant. The following table shows certain values of t and M.

t	0	21	63
M		80	

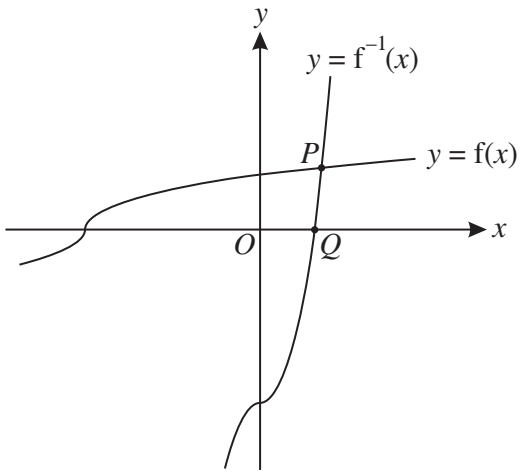
(i) In either order,

(a) find the values missing from the table, **[3]**

(b) determine the value of k. **[2]**

(ii) Find the rate at which the mass is increasing when $t = 21$. **[3]**

6



The function f is defined for all real values of x by

$$f(x) = \sqrt[3]{\tfrac{1}{2}x + 2}.$$

The graphs of $y = f(x)$ and $y = f^{-1}(x)$ meet at the point P, and the graph of $y = f^{-1}(x)$ meets the x-axis at Q (see diagram).

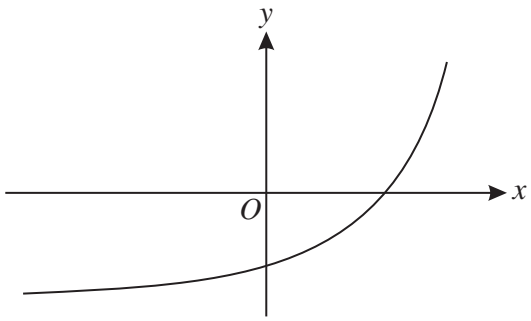
(i) Find an expression for $f^{-1}(x)$ and determine the x-coordinate of the point Q. **[3]**

(ii) State how the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are related geometrically, and hence show that the x-coordinate of the point P is the root of the equation

$$x = \sqrt[3]{\tfrac{1}{2}x + 2}.$$ **[2]**

(iii) Use an iterative process, based on the equation $x = \sqrt[3]{\tfrac{1}{2}x + 2}$, to find the x-coordinate of P, giving your answer correct to 2 decimal places. **[4]**

7



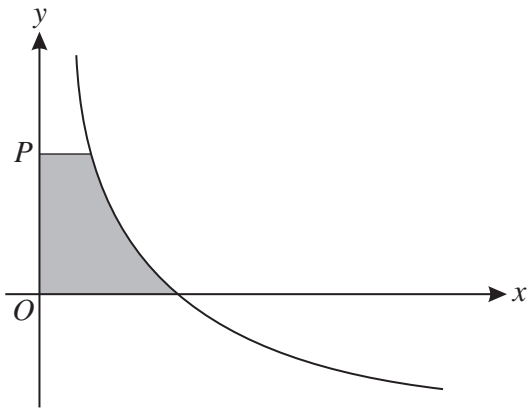
The diagram shows the curve $y = e^{kx} - a$, where k and a are constants.

(i) Give details of the pair of transformations which transforms the curve $y = e^{x}$ to the curve $y = e^{kx} - a$. **[3]**

(ii) Sketch the curve $y = \left|e^{kx} - a\right|$. **[2]**

(iii) Given that the curve $y = \left|e^{kx} - a\right|$ passes through the points $(0, 13)$ and $(\ln 3, 13)$, find the values of k and a. **[4]**

8



The diagram shows the curve with equation

$$y = \frac{6}{\sqrt{x}} - 3.$$

The point P has coordinates $(0, p)$. The shaded region is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$. The shaded region is rotated completely about the y-axis to form a solid of volume V.

(i) Show that $V = 16\pi\left(1 - \dfrac{27}{(p+3)^3}\right)$. **[6]**

(ii) It is given that P is moving along the y-axis in such a way that, at time t, the variables p and t are related by

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \tfrac{1}{3}p + 1.$$

Find the value of $\dfrac{\mathrm{d}V}{\mathrm{d}t}$ at the instant when $p = 9$. **[4]**

9 **(i)** By first expanding $\cos(2\theta + \theta)$, prove that

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta.$$ **[4]**

(ii) Hence prove that

$$\cos 6\theta \equiv 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1.$$ **[3]**

(iii) Show that the only solutions of the equation

$$1 + \cos 6\theta = 18\cos^2\theta$$

are odd multiples of 90°. **[5]**

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1 (i)	Obtain integral of form ke^{-2x}	M1		any constant k different from 8
	Obtain $-4e^{-2x}$	A1		or (unsimplified) equiv
(ii)	Obtain integral of form $k(4x+5)^7$	M1		any constant k
	Obtain $\frac{1}{28}(4x+5)^7$	A1		in simplified form
	Include $\ldots + c$ at least once	B1		in either part
			5	

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2 (i)	Form expression involving attempts at y values and addition	M1		with coeffs 1, 4 and 2 present at least once
	Obtain $k(\ln 4 + 4\ln 6 + 2\ln 8 + 4\ln 10 + \ln 12)$	A1		any constant k
	Use value of k as $\frac{1}{3}\times 2$	A1		or unsimplified equiv
	Obtain 16.27	A1	**4**	or 16.3 or greater accuracy (16.27164…)
(ii)	State 162.7 or 163	B1√	**1**	following their answer to **(i)**, maybe rounded
			5	

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3 (i)	Attempt use of identity for $\tan^2\theta$	M1		using $\pm\sec^2\theta \pm 1$; or equiv
	Replace $\dfrac{1}{\cos\theta}$ by $\sec\theta$	B1		
	Obtain $2(\sec^2\theta - 1) - \sec\theta$	A1	**3**	or equiv
(ii)	Attempt soln of quadratic in $\sec\theta$ or $\cos\theta$	M1		as far as factorisation or substitution in correct formula
	Relate $\sec\theta$ to $\cos\theta$ and attempt at least one value of θ	M1		may be implied
	Obtain 60°, 131.8°	A1		allow 132 or greater accuracy
	Obtain 60°, 131.8°, 228.2°, 300°	A1	**4**	allow 132, 228 or greater accuracy; and no others between 0° and 360°
			7	

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4 (i)	Obtain derivative of form $kx(4x^2+1)^4$	M1		any constant k
	Obtain $40x(4x^2+1)^4$	A1		or (unsimplified) equiv
	State $x = 0$	A1√	**3**	and no other; following their derivative of form $kx(4x^2+1)^4$
(ii)	Attempt use of quotient rule	M1		or equiv
	Obtain $\dfrac{2x\ln x - x^2.\frac{1}{x}}{(\ln x)^2}$	A1		or equiv
	Equate to zero and attempt solution	M1		as far as solution involving e
	Obtain $e^{\frac{1}{2}}$	A1	**4**	or exact equiv; and no other; allow from ± (correct numerator of derivative)
			7	

5 (i) State 40 B1
Attempt value of k using 21 and 80 M1 or equiv
Obtain $40e^{21k} = 80$ and hence 0.033 A1 or equiv such as $\frac{1}{21}\ln 2$
Attempt value of M for $t = 63$ M1 using established formula or using exponential property
Obtain 320 A1 **5** or value rounding to this

(ii) Differentiate to obtain $ce^{0.033t}$ or $40ke^{kt}$ M1 any constant c different from 40
Obtain $40 \times 0.033e^{0.033t}$ A1√ following their value of k
Obtain 2.64 A1 **3** allow 2.6 or 2.64 ± 0.01 or greater accuracy (2.64056…)

8

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6 (i) Attempt correct process for finding inverse M1 maybe in terms of y so far
Obtain $2x^3 - 4$ A1 or equiv; in terms of x now
State $\sqrt[3]{2}$ or 1.26 B1 **3**

(ii) State reflection in $y = x$ B1 or clear equiv
Refer to intersection of $y = x$ and $y = f(x)$
and hence confirm $x = \sqrt[3]{\frac{1}{2}x + 2}$ B1 **2** AG; or equiv

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(iii) Obtain correct first iterate B1
Show correct process for iteration M1 with at least one more step
Obtain at least 3 correct iterates in all A1 allowing recovery after error
Obtain 1.39 A1 **4** following at least 3 steps; answer required to exactly 2 d.p.

[0 → 1.259921 → 1.380330 → 1.390784 → 1.391684
1 → 1.357209 → 1.388789 → 1.391512 → 1.391747
1.26 → 1.380337 → 1.390784 → 1.391684 → 1.391761
1.5 → 1.401020 → 1.392564 → 1.391837 → 1.391775
2 → 1.442250 → 1.396099 → 1.392141 → 1.391801]

9

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7 (i) Refer to stretch and translation M1 in either order; allow here informal terms
State stretch, factor $\frac{1}{k}$, in x direction A1 or equiv; now with correct terminology
State translation in negative y direction by a A1 **3** or equiv; now with correct terminology
[SC: If M0 but one transformation completely correct – B1]

(ii) Show attempt to reflect negative part in x-axis M1 ignoring curvature
Show correct sketch A1 **2** with correct curvature, no pronounced 'rounding' at x-axis and no obvious maximum point

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(iii) Attempt method with $x = 0$ to find value of a M1 ... other than (or in addition to) value –12
Obtain $a = 14$ A1 and nothing else
Attempt to solve for k M1 using any numerical a with sound process
Obtain $k = 3$ A1 **4**

9

8 (i)	Attempt to express x or x^2 in terms of y	M1		
	Obtain $x^2 = \dfrac{1296}{(y+3)^4}$	A1		or (unsimplified) equiv
	Obtain integral of form $k(y+3)^{-3}$	M1		any constant k
	Obtain $-432\pi(y+3)^{-3}$ or $-432(y+3)^{-3}$	A1		or (unsimplified) equiv
	Attempt evaluation using limits 0 and p	M1		for expression of form $k(y+3)^{-n}$ obtained from integration attempt; subtraction correct way round
	Confirm $16\pi(1-\dfrac{27}{(p+3)^3})$	A1	**6**	AG; necessary detail required, including appearance of π prior to final line
(ii)	State or obtain $\dfrac{\mathrm{d}V}{\mathrm{d}p} = 1296\pi(p+3)^{-4}$	B1		or equiv; perhaps involving y
	Multiply $\dfrac{\mathrm{d}p}{\mathrm{d}t}$ and attempt at $\dfrac{\mathrm{d}V}{\mathrm{d}p}$	*M1		algebraic or numerical
	Substitute $p = 9$ and attempt evaluation	M1		dep *M
	Obtain $\frac{1}{4}\pi$ or 0.785	A1	**4**	or greater accuracy
			10	

9 (i)	State $\cos 2\theta\cos\theta - \sin 2\theta\sin\theta$	B1		
	Use at least one of $\cos 2\theta = 2\cos^2\theta - 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$	B1		
	Attempt to express in terms of $\cos\theta$ only	M1		using correct identities for $\cos 2\theta$, $\sin 2\theta$ and $\sin^2\theta$
	Obtain $4\cos^3\theta - 3\cos\theta$	A1	**4**	AG; necessary detail required
(ii)	Either: State or imply $\cos 6\theta = 2\cos^2 3\theta - 1$	B1		
	Use expression for $\cos 3\theta$ and attempt expansion	M1		for expression of form $\pm 2\cos^2 3\theta \pm 1$
	Obtain $32c^6 - 48c^4 + 18c^2 - 1$	A1	**3**	AG; necessary detail required
	Or: State $\cos 6\theta = 4\cos^3 2\theta - 3\cos 2\theta$	B1		maybe implied
	Express $\cos 2\theta$ in terms of $\cos\theta$ and attempt expansion	M1		for expression of form $\pm 2\cos^2\theta \pm 1$
	Obtain $32c^6 - 48c^4 + 18c^2 - 1$	A1	**(3)**	AG; necessary detail required
(iii)	Substitute for $\cos 6\theta$	*M1		with simplification attempted
	Obtain $32c^6 - 48c^4 = 0$	A1		or equiv
	Attempt solution for c of equation	M1		dep *M
	Obtain $c^2 = \frac{3}{2}$ and observe no solutions	A1		or equiv; correct work only
	Obtain $c = 0$, give at least three specific angles and conclude odd multiples of 90	A1	**5**	AG; or equiv; necessary detail required; correct work only
			12	